

# On Hausdorff spaces with a dense set of isolated points

NATHAN CARLSON\*

60 W. Olsen Rd, Thousand Oaks, CA 91360 USA

`ncarlson@callutheran.edu`

In [1] Arhangel'skii introduced the notion of a  $\tau$ -twister for a cardinal  $\tau$ , a binary operation on a topological space with certain properties, and used this to show that a power homogeneous Hausdorff space with a dense set of isolated points has countable pseudocharacter. We introduce a related notion, a  $\tau_c$ -twister, to show that in fact such spaces have countable *closed* pseudocharacter. We also show power homogeneous Hausdorff spaces with a dense set of isolated points have countable  $\pi$ -character and countable tightness. The former is a consequence of a result of Ridderbos.

We also show that if  $X$  is any Hausdorff space with dense set of isolated points then  $|X| \leq 2^{wL(X)ot(X)\psi_c(X)}$ , improving results of Bella, Carlson, and Gotchev in [2]. (The  $o$ -tightness  $ot(X)$  has the property  $ot(X) \leq \min\{t(X), c(X)\}$  for any space  $X$ ). It follows that a weakly Lindelöf power homogeneous Hausdorff space with a dense set of isolated points has cardinality at most the cardinal of the continuum. This improves a result of Arhangel'skii.

Finally we give the following characterization: Let  $X$  be a countable Hausdorff space. Then  $X$  has a dense set of isolated points if and only if  $X$  has a  $\pi$ -base consisting of elements with feebly compact closure.

## References

- [1] Arhangel'skii A. V., *Homogeneity of powers of spaces and the character*, Proc. Amer. Math. Soc. 133 (2005), no. 7, 2165–2172.
- [2] A. Bella, N. Carlson, I. Gotchev, *On spaces with a  $\pi$ -base with elements with compact closure*, Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat., 118, 26 (2024). <https://doi.org/10.1007/s13398-023-01526-3>

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