

Summing and interleaving T- and TB-sequences

ANNA GIORDANO BRUNO*

University of Udine, Italy
anna.giordanobruno@uniud.it

Protasov and Zelenyuk [3] defined a sequence $\mathbf{u} = (u_n)$ in an abelian group G to be a *T-sequence* if there exists a Hausdorff group topology τ on G such that $u_n \rightarrow 0$ in (G, τ) . Similarly, $\mathbf{u}$ is a *TB-sequence* if such a group topology τ can be chosen precompact [1].

Motivated by a question of Protasov and Zelenyuk [4] and by a paper of Gabrielyan [2], we study the following problem: given two T-sequences (respectively, TB-sequences), $\mathbf{u} = (u_n)$ and $\mathbf{v} = (v_n)$ in an abelian group G , when are their sum $\mathbf{u} + \mathbf{v} = (u_n + v_n)$ and their interleaving sequence $\mathbf{u} \sqcup \mathbf{v} = (w_n)$, with $w_{2n} = u_n$ and $w_{2n+1} = v_n$, again T-sequences (respectively, TB-sequences)?

For $G = \mathbb{Z}$ and for b -bounded arithmetic sequences $\mathbf{u} = (u_n)$ and $\mathbf{v} = (v_n)$ (i.e., the ratios u_{n+1}/u_n and v_{n+1}/v_n are positive integers and form bounded sequences), we prove that $\mathbf{u} \sqcup \mathbf{v}$ is always a T-sequence, and we characterize when it is a TB-sequence. In the special case where $\mathbf{u} = (a^n)$ and $\mathbf{v} = (b^n)$ for some integers a, b , we give a complete description of when $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} \sqcup \mathbf{v}$ are T-sequences or TB-sequences. As a consequence, we obtain a new family of minimally almost periodic group topologies on $\mathbb{Z}$.

References

- [1] G. Barbieri, D. Dikranjan, C. Milan, H. Weber, *Answer to Raczkowski's question on convergent sequences of integers*, Topology Appl. 132 (2002), 89–101.
- [2] S. Gabrielyan, *Topologies on groups determined by sets of convergent sequences*, J. Pure Appl. Algebra 217 (2013), 786–802.
- [3] I. Protasov, E. Zelenyuk, *Topologies on groups determined by sequences*, VNTL Publishers, Lviv, 1999.
- [4] I. V. Protasov, E. G. Zelenyuk, *Topologies on $\mathbb{Z}$ determined by sequences: seven open problems*, Mat. Stud. 12 (1999), no. 1, 111.

*This is joint work with Dikran Dikranjan (University of Udine).