

An introduction to Shape Space

KIM KENOBI

Department of Mathematics, Aberystwyth University, Penglais, Aberystwyth, SY233BZ
kik10@aber.ac.uk

Using the ideas of Dryden and Mardia (2016), I develop the concept of the shape space of k landmarks in m dimensions, which is denoted Σ_m^k . Given a configuration matrix, X (a $k \times m$ matrix), the **shape** of X is defined as the information that remains when location, scale and rotational effects are removed (Kendall, 1977). We consider equivalence classes of all configurations that can be obtained from X by translation, scaling and rotation. After centring and scaling X to have unit centroid size, we obtain a **pre-shape** which lies on a hypersphere of radius 1. We then consider all possible rotations (i.e. post-multiplication by $\Gamma \in SO(m)$, the group of special orthogonal $m \times m$ matrices, and define the shape of X , written $[X]$, as

$$[X] = \{Z_X \Gamma : \Gamma \in SO(m)\},$$

where Z_X is the centred and scaled version of X , and hence lies on the pre-shape sphere.

Defined this way, it is possible to define a metric on Σ_m^k , making the shape space a Riemannian manifold. I consider the mean shape of a set of shapes as well as Procrustes registration of one shape onto another and discuss tangent space to shape space. I finish with a biological example (the coordinates of 19 landmarks from photos of cichlid fish in Lakes Malawi, Tanganyika and Victoria) where I illustrate how standard multivariate statistical tools (principal component analysis and linear discriminant analysis) enable us to use tangent space to visualise the shape changes in shape space.

References

- [1] Dryden, I.L. and Mardia, K.V. (2016). Statistical Shape Analysis, with Applications in R. Second Edition. Wiley, Chichester.
- [2] Kendall, D. G. (1997). The diffusion of shape. *Advances in Applied Probability*, 9: 428-430.