

Exploring a notion of betweenness

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The notion of a point c being *between points a and b* in a set X is a natural one, assuming that there is a structure in place on X to support such a phenomenon. In such a case, it sets up a natural ternary relation $I(\cdot, \cdot, \cdot)$ where we understand $I(a, c, b)$ as meaning the previous line's assertion. To reflect this intuition, we impose three basic conditions on such a ternary relation to yield a *betweenness relation*.

Classically then, a betweenness relation on a set arises naturally from some pre-existing structure on the set. Typical examples include orderings, graphs, vector operations, metrics and topologies. We explore the nature and properties of two betweenness relations arising from metric spaces via their core triangle (in)equality condition. The first of these was introduced by Karl Menger in 1928 whereby a point c is said to be *between points a and b* in a metric space (X, ρ) if $\rho(a, b) = \rho(a, c) + \rho(c, b)$. The second, and contrasting, betweenness relation is the ultrametric version where we declare c to be between a and b if $\rho(a, b) = \max\{\rho(a, c), \rho(c, b)\}$. We note that this version aligns strongly with subcontinuum betweenness amongst hereditarily indecomposable continua, a betweenness relation deriving from the more recent antecedent structure of *road system* introduced by Paul Bankston in 2013.

In the spirit of the Galway Topology Colloquia series, this talk will be a tour through our early-stage investigations, offering some glimpses of further work done and more still to do!

References

- [1] Karl Menger, *Untersuchungen über allgemeine Metrik*, Math. Ann. 100 (1928), 75–163

*This is joint work with Paul Bankston (Marquette University, Wisconsin).